BAULKHAM HILLS HIGH SCHOOL

2018

YEAR 12 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
  Black pen is preferred
- Write your NESA# and Teacher's name on your answer booklet
- Board-approved calculators may be used
- A reference sheet is provided at the back of this paper
- In Questions 11 – 14, show relevant mathematical reasoning and/or calculations
- Marks may be deducted for careless or badly arranged work

Total marks: 70

Section I – 10 marks (pages 2 – 5)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 6 – 12)

- Attempt Questions 11 – 14
- Allow about 1 hour 45 minutes for this section

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10

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1 If the acute angle between the lines $y = 2x + 5$ and $mx - y - 3 = 0$ is $45°$, the possible values of *m* are

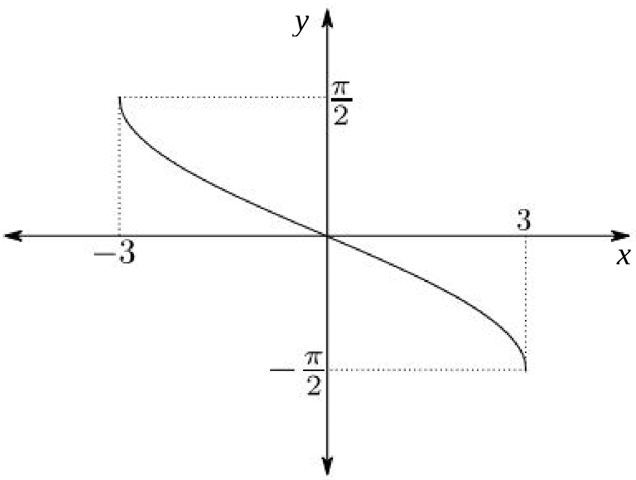
(A) $\frac{1}{3}$ and -3

(B) $\frac{1}{2}$ and -2

(C) $-\frac{1}{2}$ and 2

(D) $-\frac{1}{3}$ and 3

2



The correct function for the graph above is

(A) $y = -\sin^{-1}\left(\frac{x}{3}\right)$

(B) $y = -\sin^{-1}3x$

(C) $y = \frac{\pi}{2} - \cos^{-1}\left(\frac{x}{3}\right)$

(D) $y = \frac{\pi}{2} - \cos^{-1}3x$

3 When the polynomial $P(x)$ is divided by $(x+3)(x-4)$ the remainder is $(3x+2)$.

What is the remainder when $P(x)$ is divided by $(x-4)$?

(A) -10

(B) -7

(C) 11

(D) 14

4 A particle is moving along the x-axis such that its velocity, v, at position, x, is given by $v=\sqrt{10x-x^2}$.

What is the acceleration of the particle when $x = 1$?

(A) $\frac{4}{3}$

(B) $\frac{8}{3}$

(C) 3

(D) 4

5 Which of the following is an asymptote of the curve $y=\frac{x^2-1}{x}$?

(A) $y=x$

(B) $y=0$

(C) $y=1$

(D) $x=1$

6 Which of the following are true for all real values of *x*?

I $\sin\left(\frac{\pi}{2}+x\right)=\cos\left(\frac{\pi}{2}-x\right)$

II $2+2\sin x-\cos^2 x \geq 0$

III $\sin\left(x+\frac{3\pi}{2}\right)=\cos(\pi-x)$

IV $\sin x\cos x \leq \frac{1}{4}$

(A) **I** and **II**

(B) **II** and **III**

(C) **II** and **IV**

(D) **III** and **IV**

7 The letters of the word **PERSEVERE** are arranged in a row. The number of different arrangements that are possible if all of the four **E**'s remain together are

(A) $6!$

(B) $\frac{6!}{2!}$

(C) $\frac{9!}{2!}$

(D) $\frac{9!}{4!2!}$

8 If a focal chord of the parabola $x^2 = 4ay$ cuts the parabola at two distinct points (x_1,y_1) and (x_2,y_2) , then;

(A) $x_1\, x_2 = a^2$

(B) $y_1\, y_2 = a^2$

(C) $x_1(x_2)^2 = a^2$

(D) $y_1(y_2)^2 = a^2$

9 Consider two functions

$$f(x) = a - x^2$$
$$g(x) = x^4 - a$$

For precisely which values of $a > 0$ is the area of the region bounded by the x-axis and the curve $y = f(x)$ bigger than the area of the region bounded by the x-axis and the curve $y = g(x)$?

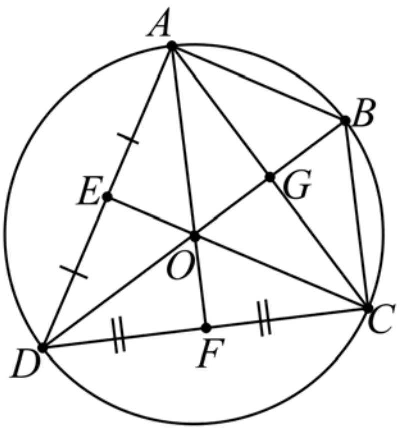
(A) $a > 1$

(B) $a > \frac{6}{5}$

(C) $a > \left(\frac{4}{3}\right)^{\frac{3}{2}}$

(D) $a > \left(\frac{6}{5}\right)^4$

10 *A*, *B*, *C* and *D* are concyclic points on a circle centre *O*. *E*, *F* and *G* are points on the chords *AD*, *CD* and *AC* respectively, such that *AF*, *CE* and *DG* are concurrent at *O*, and *D*, *O*, *G* and *B* are collinear. *AE* = *DE* and *CF* = *DF*.



Which of the following statements is **NOT** true?

(A) $\angle BAD = 90°$

(B) $\angle AGD = 90°$

(C) *ABCO* is a cyclic quadrilateral

(D) $\Delta\, ABO$ is similar to $\Delta\, ACD$

END OF SECTION I

Section II

90 marks
Attempt Questions 11 – 14
Allow about 1 hour 45 minutes for this section

Answer each question on the appropriate page in the answer booklet. Extra paper is available, write your NESA# on any extra paper you use.

In Questions 11 to 14, your responses should include relevant mathematical reasoning and/or calculations.

Marks

<u>**Question 11**</u> ***(15 marks)*** *Start on the page labelled Question 11 in your answer booklet*

(a) Find $\int \cos^2 x \, dx$ *2*

(b) Solve $\frac{2x}{5-x} \geq 1$ *3*

(c) Differentiate $\sin^{-1}(x^2)$ *2*

(d) Express $15\cos x - 8\sin x$ in the form $A\cos(x + \alpha)$,where $0 \leq \alpha \leq \frac{\pi}{2}$ *2*

(e) You are given that in the expansion of $(a + bx)^5$, the constant term is 32 and the coefficient of x^3 is -1080. *3*

Find the values of *a* and *b*.

(f) A particle moves so that its distance, *x* centimetres, from a fixed point *O* at time, *t* seconds, is $x = 4\sin 2t$.

(i) Show that the particle is moving in Simple Harmonic Motion *2*

(ii) What is the frequency of the particle's motion, in oscillations per second? *1*

Marks

<u>Question 12</u> ***(15 marks)*** *Start on the page labelled Question 12 in your answer booklet*

(a) Evaluate $\lim_{x \to 2} \frac{\sin(2 - x)}{(x - 2)(x + 3)}$ *2*

(b) Celeste and Michelle are playing a table tennis match. The winner of the match is the first player to win three games.

The probability that Celeste wins a game is 0.55, games cannot be drawn.

Find, correct to two decimal places, the probability that

(i) Celeste wins the match in three games. *1*

(ii) Celeste wins the match. *2*

(c) Use mathematical induction to prove that *3*

$$\frac{2}{1 \times 2 \times 3} + \frac{2}{2 \times 3 \times 4} + \frac{2}{3 \times 4 \times 5} + \dots + \frac{2}{n(n + 1)(n + 2)} = \frac{1}{2} - \frac{1}{(n + 1)(n + 2)}$$

for all integers $n \geq 1$

(d) Using the expansion of $(1 + x)^n$, find the value of

(i) $\binom{n}{0} + 2\binom{n}{1} + 4\binom{n}{2} + \dots + 2^n\binom{n}{n}$ *1*

(ii) $\binom{n}{1} - 2\binom{n}{2} + 3\binom{n}{3} - \dots + (-1)^{n-1}n\binom{n}{n}$ *2*

Question 12 continues on page 8

Marks

Question 12 (continued)

(e) It is known that a root of the equation $e^x - 2x^2 = 0$ exists in the interval $0.6 < x < 2.4$

(i) Use one application of Newton's method to find a three decimal place approximation to a root of the equation $e^x - 2x^2 = 0$, using $x_0 = 1.5$ as a first approximation. *2*

(ii) Copy and complete, correct to two decimal places, the following table of values for $P(x) = e^x - 2x^2$ *1*

x	2.0	2.1	2.2	2.3	2.4
$P(x)$					

(iii) Hence, without further calculation, explain why $x_0 = 2.3$ would have been a less suitable first approximation for the root of $P(x) = 0$ that lies in the interval $0.6 < x < 2.4$ *1*

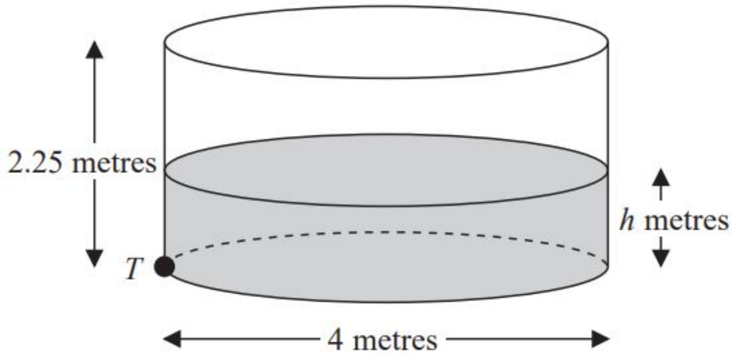
End of Question 12

Marks

Question 13 ***(15 marks)*** *Start on the page labelled Question 13 in your answer booklet*

(a) A cylindrical tank has diameter 4 metres and height 2.25 metres. Water is flowing into the tank at a rate of $\frac{2\pi}{5}$ m^3/min.

There is a tap at a point *T* at the base of the tank. When the tap is opened, water leaves the tank at a rate of $\frac{\pi}{5}\sqrt{h}$ m^3/min, where *h* is the height of the water in metres.



(i) Show that at time *t* minutes after the tap has opened, the volume of water in the tank satisfies the differential equation *1*

$$\frac{dV}{dt} = \frac{\pi(2-\sqrt{h})}{5}$$

(ii) Show that at time *t* minutes after the tap has opened, the height of the water in the tank satisfies the differential equation *2*

$$\frac{dh}{dt} = \frac{2-\sqrt{h}}{20}$$

(iii) When the tap is opened the height of the water is 0.16 metres. The time taken to fill the tank to a height of 2.25 metres can be calculated using *3*

$$t = \int_{0.16}^{2.25} \frac{20}{2-\sqrt{h}}\,dh \qquad \text{(Do NOT prove this)}$$

Using the substitution $h=(2-x)^2$, where $0<x<2$, find the time taken to fill the tank, correct to the nearest minute.

Question 13 continues on page 10

Marks

<u>Question 13</u> (continued)

(b) Two hundred rabbits in a region with an estimated population of 200 000 rabbits have a highly contagious disease.

The disease is known to spread at the weekly rate of 1% of the remaining healthy rabbits such that

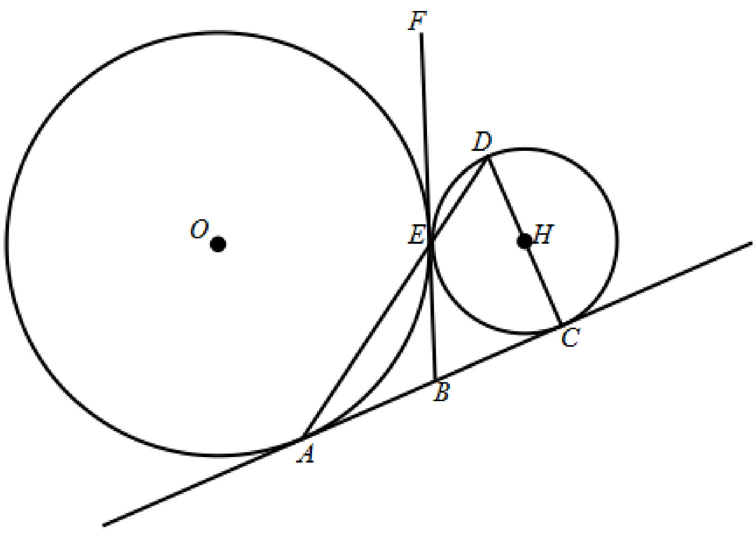
$$\frac{dP}{dt} = 0.01(200\,000 - P)$$

where P is the number of infected rabbits after t weeks.

(i) Show that $P = 200\,000 - 199\,800e^{-0.01t}$ satisfies both the differential equation and the initial conditions. *2*

(ii) How many days does it take for half of the rabbit population to become infected? *3*

(c) *4*



In the diagram, two circles with centres O and H touch externally at E.
The common tangent at E meets another common tangent AC at B.
CD is a diameter of the smaller circle.

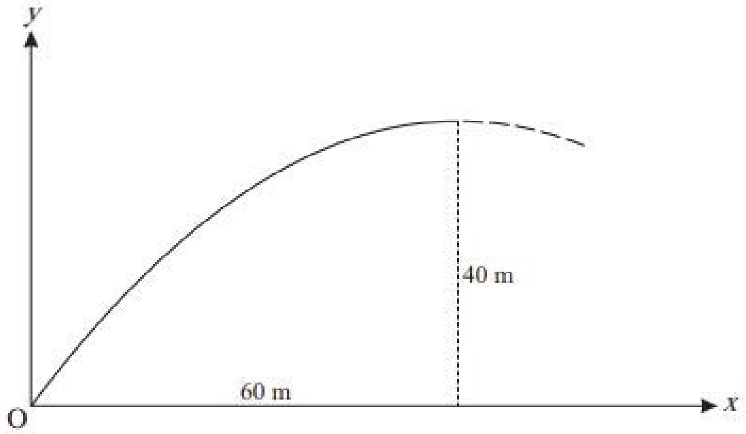
Copy the diagram into your answer booklet and prove that A, E and D are collinear.

End of Question 13

Marks

Question 14 ***(15 marks)*** *Start on the page labelled Question 14 in your answer booklet*

(a) A small firework is fired at ground level with initial speed $V\ \text{ms}^{-1}$ at an angle of θ to the horizontal. The highest point reached by the firework is at a horizontal distance of 60 metres from the point of projection and a vertical distance of 40 metres above the ground.



Neglecting the effects of air resistance, the equations describing the motions of the firework are

$$x = Vt\cos\theta$$
$$y = Vt\sin\theta - 4.9t^2$$

where *t* is the time in seconds after the firework is projected. Do NOT prove these equations.

It is known that the initial horizontal velocity of the firework is $21\ \text{ms}^{-1}$

(i) Calculate the time for the firework to reach its highest point, correct to two decimal places *2*

(ii) Show that the initial vertical velocity is $28\ \text{ms}^{-1}$ *2*

When the firework is at its highest point it explodes into several parts. Two of these parts initially continue to travel horizontally, one with the original horizontal speed of $21\ \text{ms}^{-1}$ and the other with a quarter of this speed.

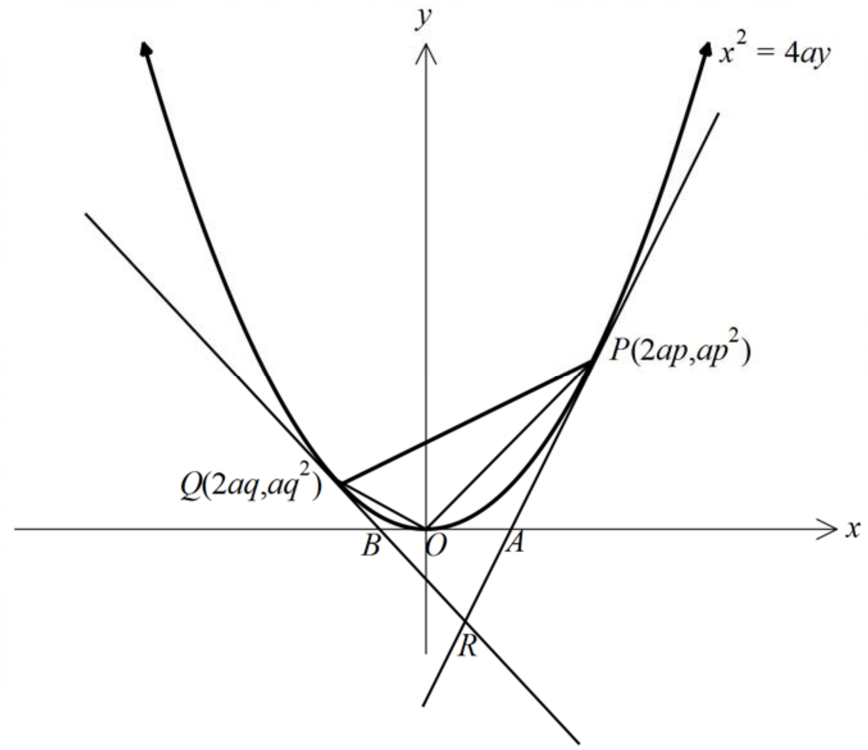
(iii) State why the two parts are always at the same height as one another above the ground *1*

(iv) Find the distance between the two parts of the firework when they hit the ground, correct to the nearest metre. *2*

Question 14 continues on page 12

Question 14 (continued)

(b) The points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$, where $p > 0$ and $q < 0$, and $|p| > |q|$, lie on the parabola $x^2 = 4ay$.



(i) Write down the equations of the tangents to the parabola at P and Q *1*

(ii) The tangents to the parabola at P and Q meet at R. *2*
Show that R has coordinates $\{a(p+q), apq\}$

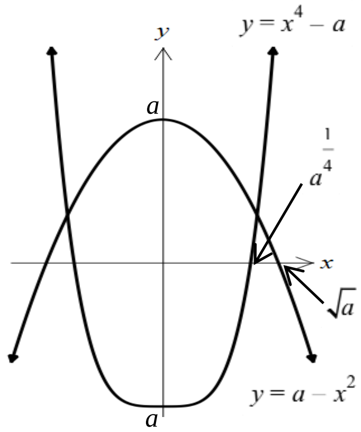
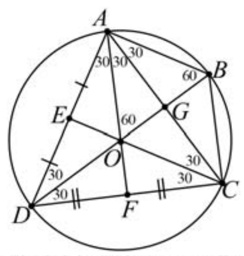
(iii) The tangents at P and Q meet the x-axis at A and B respectively. *2*
Show that the area of ΔABR is $\frac{1}{2}a^2pq(q-p)$

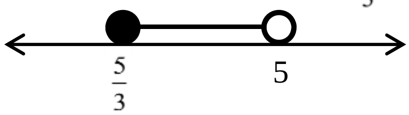
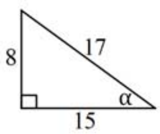
(iv) Prove that the area of ΔOPQ is twice the area of ΔABR *3*

End of paper

BAULKHAM HILLS HIGH SCHOOL
YEAR 12 EXTENSION 1 TRIAL 2018 SOLUTIONS

Solution	Marks	Comments
SECTION I		
1. A – $y = 2x + 5 \Rightarrow m_1 = 2$ $\quad mx - y - 3 = 0 \Rightarrow m_2 = m$ $\tan 45° = \left\lvert \frac{2-m}{1+2m} \right\rvert$ $1 = \left\lvert \frac{2-m}{1+2m} \right\rvert$ $\lvert 2-m \rvert = \lvert 1+2m \rvert$ $2 - m = 1 + 2m$ or $-(2-m) = 1 + 2m$ $-3m = -1$ $\quad m - 2 = 1 + 2m$ $m = \frac{1}{3}$ $\quad m = -3$	1	
2. A – Domain : $-1 \le ax \le 1$ $-\frac{1}{a} \le x \le \frac{1}{a}$ $\therefore \frac{1}{a} = 3$ $a = \frac{1}{3}$ Curve is either $\sin^{-1}f(x)$ flipped upside down i.e $-\sin^{-1}f(x)$ or $\cos^{-1}f(x)$ shifted down $\frac{\pi}{2}$ i.e. $\cos^{-1}f(x) - \frac{\pi}{2}$ $\therefore$ the correct function is $y = -\sin^{-1}\left(\frac{x}{3}\right)$	1	
3. D – $P(x) = (x+3)(x-4)Q(x) + (3x+2)$ $P(4) = 0 + 3(4) + 2$ $= 14$	1	
4. D – $\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$ $= \frac{d}{dx}\left(5x - \frac{1}{2}x^2\right)$ $= 5 - x$ when $x = 1$; $\ddot{x} = 5 - 1$ $= 4$	1	
5. A - $y = \frac{x^2 - 1}{x}$ $= x - \frac{1}{x}$ asymptotes are $y = x$ and $x = 0$	1	
6. B - **I**: Let $x = 0$, $\sin\frac{\pi}{2} = \cos\frac{\pi}{2}$ ✗ **II**: $2 + 2\sin x - \cos^2 x = 2 + 2\sin x - 1 + \sin^2 x$ ✓ $= \sin^2 x + 2\sin x + 1$ $= (\sin x + 1)^2 \ge 0$ **III**: $\sin\left(x + \frac{3\pi}{2}\right) = \cos\left\{\frac{\pi}{2} - \left(x + \frac{3\pi}{2}\right)\right\}$ ✓ $= \cos(\pi - x)$ **IV**: $\sin x \cos x = \frac{1}{2}\sin 2x$ ✗ $\le \frac{1}{2}$ Thus the two that are correct are **II** and **III**	1	
7. B – The four **E**’s are now treated as one letter, so the question becomes how many arrangements of **PRSVR(EEEE)** Ways $= \frac{6!}{2!}$	1	

Solution	Marks	Comments
8. B - $m_{SP} = m_{SQ}$ $\frac{y_1 - a}{x_1} = \frac{y_2 - a}{x_2}$ $x_2 y_1 - a x_2 = x_1 y_2 - a x_1$ $\frac{x_1^2 x_2}{4a} - a x_2 = \frac{x_1 x_2^2}{4a} - a x_1$ $x_1 x_2 (x_1 - x_2) = 4a^2 (x_2 - x_1)$ $x_1 x_2 = -4a^2$ $(x_1 x_2)^2 = 16a^4$ $4ay_1 \times 4ay_2 = 16a^4$ $y_1 y_2 = a^2$	**1**	
9. D – $\int_0^{\sqrt{a}} a - x^2 \, dx > -\int_0^{\sqrt{a}} x^4 - a \, dx$ $\left[ax - \frac{1}{3}x^3 \right]_0^{a^{\frac{1}{2}}} > -\left[\frac{1}{5}x^5 - ax \right]_0^{a^{\frac{1}{2}}}$ $a^{\frac{3}{2}} - \frac{1}{3}a^{\frac{3}{2}} > -\frac{1}{5}a^{\frac{5}{4}} + a^{\frac{5}{4}}$ $\frac{2}{3}a^{\frac{3}{2}} - \frac{4}{5}a^{\frac{5}{4}} > 0$ $a^{\frac{5}{4}}\left(\frac{2}{3}a^{\frac{1}{4}} - \frac{4}{5} \right) > 0$ By definition $a^{\frac{5}{4}} > 0$ Thus $\frac{2}{3}a^{\frac{1}{4}} - \frac{4}{5} > 0$ $a^{\frac{1}{4}} > \frac{4}{5} \times \frac{3}{2}$ $a > \left(\frac{6}{5} \right)^4$ 	**1**	
10. C - $\angle BAD = 90°$ (∠ in a semicircle) $\angle AGD = 90°$ AF ⊥ CD , CE ⊥ AD (perp from centre bisects chord) ∴ DG ⊥ AC (altitudes are concurrent)  altitudes , medians , perpendicular bisectors are the same lines ∴ Δ ACD is equilateral $\angle BOA = 60°$ (∠ at centre"," twice ∠ at circumference) so Δ ABO also turns out to be equilateral Thus Δ ABO ||| Δ ACD *ABCO* is **NOT** a cyclic quadrilateral as opposite ∠ 's are **NOT** supplementary ∴ answer is ***C***	**1**	

SECTION II		
Solution	**Marks**	**Comments**
QUESTION 11		
11(a) $\int \cos^2 x\, dx = \frac{1}{2}\int 1 + \cos 2x\, dx$ $= \frac{1}{2}\left(x + \frac{1}{2}\sin 2x \right) + c$ $= \frac{x}{2} + \frac{1}{4}\sin 2x + c$	2	**2 marks** • Correct solution **1 mark** • Find a correct relationship between $\cos^2 x$ and $\cos 2x$
11(b) $\frac{2x}{5-x} \geq 1$ $5 - x \neq 0$ $x \neq 5$ $\frac{2x}{5-x} = 1$ $2x = 5 - x$ $3x = 5$ $x = \frac{5}{3}$  $\frac{5}{3} \leq x < 5$	3	**3 marks** • Correct graphical solution on number line or algebraic solution, with correct working **2 marks** • Bald answer • Identifies the two correct critical points via a correct method • Correct conclusion to their critical points obtained using a correct method **1 mark** • Uses a correct method • Acknowledges a problem with the denominator. **0 marks** • Solves like a normal equation , with no consideration of the denominator.
11(c) $f(x) = \sin^{-1}(x^2)$ $f'(x) = \frac{2x}{\sqrt{1-x^4}}$	2	**2 marks** • Correct solution **1 mark** • obtains $\frac{g(x)}{\sqrt{1-x^4}}$ or equivalent merit
11 (d)  $\alpha = \tan^{-1}\left(\frac{8}{15}\right)$ $= 0.4899573263...$ $15\cos x - 8\sin x$ $= 17\cos\left(x + \tan^{-1}\frac{8}{15}\right)$ OR $= 17\cos(x + 0.4899573263...)$	2	**2 marks** • Correct solution **1 mark** • Finds *A* • establishes $\alpha = \tan^{-1}\frac{8}{15}$ *Note: no penalty for rounding, if it is clear how α has been established*
11 (e) $a^5 = 32$ $a = 2$ $\binom{5}{3} a^2 (bx)^3 = -1080x^3$ $(10)(4)b^3 = -1080$ $b^3 = -27$ $b = -3$	3	**3 marks** • Correct solution **2 marks** • Finds *b* • Finds *a* and the term involving x^3 **1 mark** • Finds *a* • Finds the term involving x^3

Solution	Marks	Comments
11(f) (i) $x = 4\sin 2t$ $\dot{x} = 8\cos 2t$ $\ddot{x} = -16\sin 2t$ $= -4(4\sin 2t)$ $= -4x$ $\therefore$ particle moves in SHM as $\ddot{x} = -n^2 x$	**2**	**2 marks** • Correct solution **1 mark** • Recognises the condition for a particle to move in SHM • Correctly obtains acceleration as a function of time by differentiation
11 (f) (ii) $\ddot{x} = -4x$ $\therefore n = 2$ $f = \frac{n}{2\pi}$ $= \frac{2}{2\pi}$ $= \frac{1}{\pi}$ $\therefore$ frequency of the particle is $\frac{1}{\pi}$ oscillations/second (Hz)	**1**	**1 mark** • Correct answer
QUESTION 12		
12 (a) $\lim_{x \to 2} \frac{\sin(2-x)}{(x-2)(x+3)} = \lim_{x \to 2} \frac{\sin(2-x)}{(2-x)} \times \frac{-1}{(x+3)}$ $= 1 \times -\frac{1}{5}$ $= -\frac{1}{5}$	**2**	**2 marks** • Correct solution **1 mark** • Attempts to use the "small angle" theorem
12 (b) (i) $P(\text{Celeste wins in 3 games}) = (0.55)^3$ $= 0.166375$ $= 0.17$ (to 2 dp)	**1**	**1 mark** • Correct answer
12 (b) (ii) In order for Celeste to win, she must win the last game and two others. Michelle could win 0, 1 or 2 games $P(\text{Celeste wins}) = 0.55\left\{\binom{2}{0}(0.55)^2(0.45)^0 + \binom{3}{1}(0.55)^2(0.45) + \binom{4}{2}(0.55)^2(0.45)^2\right\}$ $= 0.593126875...$ $= 0.59$ (to 2 dp)	**2**	**2 marks** • Correct solution **1 mark** • Establishes multiple situations where Celeste wins
12 (c) When $n = 1$; $LHS = \frac{2}{1 \times 2 \times 3} = \frac{2}{6} = \frac{1}{3}$ $RHS = \frac{1}{2} - \frac{1}{2 \times 3} = \frac{1}{2} - \frac{1}{6} = \frac{1}{3}$ $\therefore$ LHS=RHS Hence the result is true for $n = 1$ Assume the result is true for $n = k$ i.e. $\frac{2}{1 \times 2 \times 3} + \frac{2}{2 \times 3 \times 4} + \frac{2}{3 \times 4 \times 5} + ... + \frac{2}{k(k+1)(k+2)} = \frac{1}{2} - \frac{1}{(k+1)(k+2)}$ Prove the result is true for $n = k + 1$ i.e. $\frac{2}{1 \times 2 \times 3} + \frac{2}{2 \times 3 \times 4} + \frac{2}{3 \times 4 \times 5} + ... + \frac{2}{(k+1)(k+2)(k+3)} = \frac{1}{2} - \frac{1}{(k+2)(k+3)}$	**3**	There are 4 key parts of the induction; 1. Proving the result true for $n = 1$ 2. Clearly stating the assumption and what is to be proven 3. Using the assumption in the proof 4. Correctly proving the required statement **3 marks** • Successfully does all of the 4 key parts **2 marks** • Successfully does 3 of the 4 key parts **1 mark** • Successfully does 2 of the 4 key parts

<table>
<tr><th>Solution</th><th>Marks</th><th>Comments</th></tr>
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12 (c)…*continued.*

PROOF:

$$\frac{2}{1\times 2\times 3}+\frac{2}{2\times 3\times 4}+\frac{2}{3\times 4\times 5}+\ldots+\frac{2}{(k+1)(k+2)(k+3)}$$
$$=\frac{2}{1\times 2\times 3}+\frac{2}{2\times 3\times 4}+\frac{2}{3\times 4\times 5}+\ldots+\frac{2}{k(k+1)(k+2)}+\frac{2}{(k+1)(k+2)(k+3)}$$
$$=\frac{1}{2}-\frac{1}{(k+1)(k+2)}+\frac{2}{(k+1)(k+2)(k+3)}$$
$$=\frac{1}{2}-\frac{(k+3)-2}{(k+1)(k+2)(k+3)}$$
$$=\frac{1}{2}-\frac{(k+1)}{(k+1)(k+2)(k+3)}$$
$$=\frac{1}{2}-\frac{1}{(k+1)(k+2)(k+3)}$$

Hence the result is true for $n = k + 1$,if it is true for $n = k$

Since the result is true for $n = 1$, then it is true for all positive integers by induction.

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12 (d) (i) $(1+x)^n=\binom{n}{0}+\binom{n}{1}x+\binom{n}{2}x^2+\ldots+\binom{n}{n}x^n$

Let $x = 2$

$$\binom{n}{0}+2\binom{n}{1}+4\binom{n}{2}+\ldots+2^n\binom{n}{n}$$
$$=(1+2)^n$$
$$=3^n$$

</td><td>1</td><td>

1 mark
- Correct solution

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12 (d) (ii) $(1+x)^n=\binom{n}{0}+\binom{n}{1}x+\binom{n}{2}x^2+\ldots+\binom{n}{n}x^n$

Differentiating both sides

$$n(1+x)^{n-1}=\binom{n}{1}+2\binom{n}{2}x+3\binom{n}{3}x^2+\ldots+n\binom{n}{n}x^{n-1}$$

Let $x=-1$

$$\binom{n}{1}-2\binom{n}{2}+3\binom{n}{3}-\ldots+(-1)^{n-1}n\binom{n}{n}$$
$$=(-1)(1-1)^{n-1}$$
$$=0$$

</td><td>2</td><td>

2 marks
- Correct solution

1 mark
- Uses $x=-1$
- Attempts to differentiate both sides of the binomial theorem

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12 (e) (i)

$$x_1=x_0-\frac{f(x_0)}{f'(x_0)}$$
$$=x_0-\frac{e^x-2x^2}{e^x-4x}$$
$$=1.5-\frac{e^{1.5}-2(1.5)^2}{e^{1.5}-4(1.5)}$$
$$=1.487939934......$$
$$=1.488 \quad \text{(correct to 3 decimal places)}$$

</td><td>2</td><td>

2 marks
- Correct solution

1 mark
- Uses Newton’s Method correctly

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12 (e) (ii)

<table>
<tr><td>x</td><td>2.0</td><td>2.1</td><td>2.2</td><td>2.3</td><td>2.4</td></tr>
<tr><td>$P(x)$</td><td>–0.61</td><td>–0.65</td><td>–0.65</td><td>–0.61</td><td>–0.50</td></tr>
</table>

</td><td>1</td><td>

1 mark
- Correctly completed table

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12 (e) (iii) From the table of values, at $x = 2.3$ the curve is increasing and is on the right hand side of the turning point. The tangent at this point would intersect the x-axis to the right of $x = 2.4$, thus producing an approximation outside the desired interval.

</td><td>1</td><td>

1 mark
- Valid explanation

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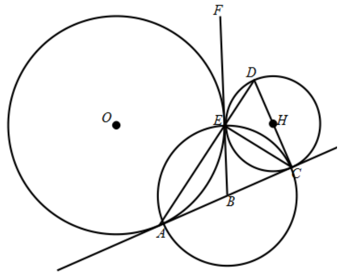
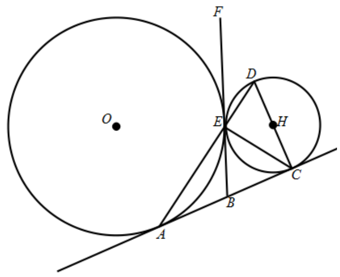
</table>

Solution	Marks	Comments
QUESTION 13		
13 (a) (i) $\frac{dV}{dt} =$ rate of water going in $-$ rate of water going out $= \frac{2\pi}{5} - \frac{\pi}{5}\sqrt{h}$ $= \frac{\pi(2-\sqrt{h})}{5}$	1	**1 mark** • Correct solution
13 (a) (ii) $V = \pi r^2 h$ $= \pi(2)^2 h$ $= 4\pi h$ $\frac{dV}{dh} = 4\pi$ $\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$ $= \frac{1}{4\pi} \times \frac{\pi(2-\sqrt{h})}{5}$ $= \frac{2-\sqrt{h}}{20}$	2	**2 marks** • Correct solution **1 mark** • finds $\frac{dV}{dh}$ • uses the chain rule to express $\frac{dh}{dt}$ as a product of other rates
13 (a) (iii) $t = \int_{0.16}^{2.25} \frac{20}{2-\sqrt{h}}\,dh$ $h = (2-x)^2$ $dh = -2(2-x)\,dx$ when $h = 0.16$: $0.16 = (2-x)^2$, $2-x = 0.4$, $x = 1.6$ when $h = 2.25$: $2.25 = (2-x)^2$, $2-x = 1.5$, $x = 0.5$ $= -20\int_{1.6}^{0.5} \frac{2(2-x)}{2-(2-x)}\,dx$ $= 40\int_{0.5}^{1.6} \frac{2-x}{x}\,dx$ $= 40\int_{0.5}^{1.6} \left(\frac{2}{x} - 1\right)dx$ $= 40[2\ln x - x]_{0.5}^{1.6}$ $= 40\left\{2\ln\left(\frac{1.6}{0.5}\right) - 1.6 + 0.5\right\}$ $= 49.05206478...$ $\therefore$ it takes 49 minutes to fill the tank	3	**3 marks** • Correct solution using the given substitution *Note: solving as an indefinite integral, then using answer to find definite integral is acceptable* **2 marks** • Correct primitive in terms of x • Correct integrand in terms of x, including the correct limits **1 mark** • Correct integrand in terms of x without the limits • Correctly finds answer using an alternative approach
13 (b) (i) $P = 200\,000 - 199\,800e^{-0.01t}$ $\frac{dP}{dt} = 1998e^{-0.01t}$ $= 0.01\{200\,000 - (200\,000 - 199\,800e^{-0.01t})\}$ $= 0.01(200\,000 - P)$ when $t = 0, P = 200\,000 - 199\,800e^0$ $= 200\,000 - 199\,800$ $= 200$	2	**2 marks** • Correct solution **1 mark** • Establishes initial population is 200 • Verifies given equation is solution to the differential equation
13 (b) (ii) $P > 100\,000$ $200\,000 - 199\,800e^{-0.01t} > 100\,000$ $199\,800e^{-0.01t} < 100\,000$ $e^{-0.01t} < \frac{100\,000}{199\,800}$ $-0.01t < \ln\left(\frac{500}{999}\right)$ $t > 100\ln\left(\frac{999}{500}\right)$ $t > 69.21466802...$ weeks $t > 484.5026762...$ days Half of the rabbit population is infected after 485 days	3	**3 marks** • Correct solution **2 marks** • Correct solution, leaving the answer in weeks • Obtains an answer of 483 or 484 days **1 mark** • Establishes an inequation, or equation, with t as the subject, using valid methods

Solution	Marks	Comments
13 (c) Join *C* and *E* $AB = BE = BC$ (tangents from external point =) so a circle with *AC* as diameter passe through *E* $\angle AEC = 90°$ ($\angle$ in semicircle, diameter *AC*) $\angle CED = 90°$ ($\angle$ in semicircle, diameter *CD*) $\angle AEC + \angle CED = 180°$ i.e. $\angle AED = 180°$ Thus *A*, *E* and *D* are collinear $90° - \angle CAD + 90 + \angle ECD = 180°$ $\angle ECD = \angle CAD$  *OR* Join *C* and *E* $AB = BE$ (tangents from external point =) $\therefore \angle BAE = \angle AEB$ ($\angle$'s opposite = sides in a Δ are =) $\angle DCA = 90°$ (radius $\perp$ tangent) $\angle DCA + \angle ADC + \angle CAD = 180°$ ($\angle$ sum ΔCAD) $90° + \angle ADC + \angle CAD = 180°$ $\angle ADC = 90 - \angle CAD$ $\angle CED = 90°$ ($\angle$ in semicircle, diameter *CD*) $\angle EDC + \angle CED + \angle ECD = 180°$ ($\angle$ sum ΔECD) $90° - \angle CAD + 90 + \angle ECD = 180°$ $\angle ECD = \angle CAD$ $\angle FED = \angle ECD$ (alternate segment theorem) Thus $\angle FED = \angle CAD$ $\angle CAD = \angle BAE$ (common $\angle$) $\therefore \angle FED = \angle AEB$ Thus *A*, *D* and *E* are collinear, as the vertically opposite $\angle$'s are equal  	4	**4 marks** • Correct solution **3 marks** • Correct solution with poor reasoning • Significant progress towards solution with good reasoning. **2 marks** • Significant progress towards solution with poor reasoning. • Progress towards solution with good reasoning. **1 mark** • Correctly uses a valid circle geometry theorem.
QUESTION 14		
14 (a) (i) $\dot{x} = V\cos\theta$ $\therefore V\cos\theta = 21$ $60 = Vt\cos\theta$ $t = \frac{60}{V\cos\theta}$ $= \frac{60}{21}$ $= 2.857142857...$ $= 2.86$ seconds (to 2 dp)	2	**2 marks** • Correct solution **1 mark** • establishes $t = \frac{60}{V\cos\theta}$
14 (a) (ii) Greatest height occurs when $\dot{y} = 0$ and $t = \frac{60}{21}$ $\dot{y} = V\sin\theta - 9.8t$ $0 = V\sin\theta - 9.8\left(\frac{60}{21}\right)$ $V\sin\theta = 28$ when $t = 0$, $\dot{y} = V\sin\theta$ $\therefore$ the initial vertical velocity is 28 ms^{-1}	2	**2 marks** • Correct solution **1 mark** • Establishes that the greatest height occurs when $\dot{y} = 0$
14 (a) (iii) Both parts have the same vertical velocity of 0 ms^{-1} at the time of explosion, so $V\sin\theta = 0$ Thus $y = -4.9t^2$ for both parts of the firework i.e. theyhave the same vertical displacement.	1	**1 mark** • Correct explanation

Solution	Marks	Comments
14 (a) (iv) Particle hits the ground when $y=-40$ *OR* Time to fall = time to rise $-40=-4.9t^2$; $t^2=\frac{400}{49}$; $t=\frac{20}{7}$ OR $t=\frac{60}{21}$ (from part (i)) $=\frac{20}{7}$ particle A: $V\cos\theta=21$, $x_A=21t$ particle B: $V\cos\theta=\frac{21}{4}$, $x_B=\frac{21}{4}t$ $x_A-x_B=21t-\frac{21}{4}t$ $=\frac{63}{4}t$ When $t=\frac{20}{7}$; $x_A-x_B=\frac{63}{4}\times\frac{20}{7}$ $=45$ ∴ the two parts of the firework land 45 metres apart	2	**2 marks** • Correct solution **1 mark** • Finds the time it takes for the two parts of the firework to hit the ground • Establishes that the distance between the two parts of the firework is given by $\frac{63}{4}t$
14 (b) (i) tangent at $P: y=px-ap^2$ tangent at $Q: y=qx-aq^2$	1	**1 mark** • Correct answers
14 (b) (ii) $y=px-ap^2$ $y=qx-aq^2$ $0=(p-q)x-a(p^2-q^2)$ $x=\frac{a(p^2-q^2)}{p-q}$ $x=\frac{a(p-q)(p+q)}{(p-q)}$ $=a(p+q)$ $\Rightarrow y=p(a(p+q))-ap^2$ $=ap^2+apq-ap^2$ $=apq$ $\therefore R\{a(p+q),apq\}$	2	**2 marks** • Correctly shows that R is the point of intersection **1 mark** • Correctly finds the x or y value of the point R. • Correctly substitutes R into one of the tangent
14 (b) (iii) tangents meet the x-axis when $y=0$ $\therefore A(ap,0)$ and $B(aq,0)$ $AB = ap - aq$ $=a(p-q)$ *Note*: as $p>0$ and $q<0$ then $apq<0$ Area $=\frac{1}{2}$bh $=\frac{1}{2}\times a(p-q)\times(-apq)$ $=\frac{1}{2}a^2pq(q-p)$	2	**2 marks** • Correctly shows the area **1 mark** • Finds the length of AB • Finds an area using $\lvert y\rvert$ coordinate of R for the perpendicular height

<table>
<tr><th>Solution</th><th>Marks</th><th>Comments</th></tr>
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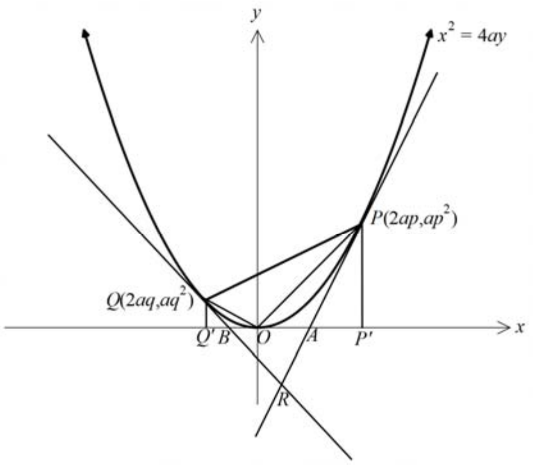
14(b) (iv)



Drop perpendiculars to the *x*-axis from *P* and *Q*, creating a trapezium *PP'Q'Q*

$$\text{Area } PP'Q'Q = \frac{1}{2} \times P'Q' \times (P'P + Q'Q) = \frac{1}{2} \times (2ap - 2aq) \times (ap^2 + aq^2) = a^2(p-q)(p^2+q^2)$$

$$\text{Area } \Delta OP'P = \frac{1}{2} \times OP' \times P'P = \frac{1}{2} \times 2ap \times ap^2 = a^2p^3$$

Similarly; replacing p with $-q$

$$\text{Area } \Delta OQ'Q = -a^2q^3$$

$$\begin{aligned}\text{Area } \Delta OPQ &= \text{Area } PP'Q'Q - \text{Area } \Delta OP'P - \text{Area } \Delta Q'Q \\ &= a^2(p-q)(p^2+q^2) - a^2p^3 + a^2q^3 \\ &= a^2\{(p-q)(p^2+q^2) - (p^3-q^3)\} \\ &= a^2\{(p-q)(p^2+q^2) - (p-q)(p^2+pq+q^2)\} \\ &= a^2(p-q)(p^2+q^2-p^2-pq-q^2) \\ &= -a^2pq(p-q) \\ &= a^2pq(q-p) \\ &= 2 \times \text{Area } \Delta ABR\end{aligned}$$

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3

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3 marks

- Correct solution

2 marks

- Finds the area of *PP'Q'Q* and the area of $\Delta OP'P$ (or $\Delta OQ'Q$)
- Finds the perpendicular distance of *O* to *PQ*
- Finds the length and equation of *PQ*

1 mark

- Finds the area of *PP'Q'Q*
- Finds the area of $\Delta OP'P$ or $\Delta OQ'Q$
- Finds the length of *PQ*
- Finds the equation of *PQ*

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